

$$\textcircled{1} \text{ (a) } \underline{a} = \frac{dv}{dt} = (3t^2 - 15)\underline{i} + (6 - 2t)\underline{j}$$

$$\text{(b) (i) } F = ma = 4[(3t^2 - 15)\underline{i} + (6 - 2t)\underline{j}]$$

$$= (12t^2 - 60)\underline{i} + (24 - 8t)\underline{j} \quad \text{as required.}$$

$$\text{(ii) } t=2 \quad \underline{F} = (12(2)^2 - 60)\underline{i} + (24 - 8(2))\underline{j}$$

$$= (48 - 60)\underline{i} + 8\underline{j}$$

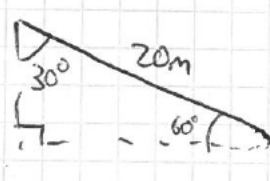
$$= -12\underline{i} + 8\underline{j}$$

$$|F| = \sqrt{12^2 + 8^2} = 4\sqrt{13}$$

$$= 14.4 \text{ N}$$

$$\textcircled{2} \text{ (a) } \frac{1}{2}mv^2 = \frac{1}{2} \times 55 \times 3^2 = 247.5 \text{ J}$$

(b)



$$h = 20 \cos 30 = 10\sqrt{3}$$

$$\Delta GPE = mg\Delta h = 55 \times 9.8 \times 10\sqrt{3}$$

$$= 5390\sqrt{3}$$

$$E_{k \text{ final}} = E_{k \text{ initial}} + \Delta GPE$$

$$= 247.5 + 5390\sqrt{3}$$

$$\frac{1}{2}mv^2 = 247.5 + 5390\sqrt{3}$$

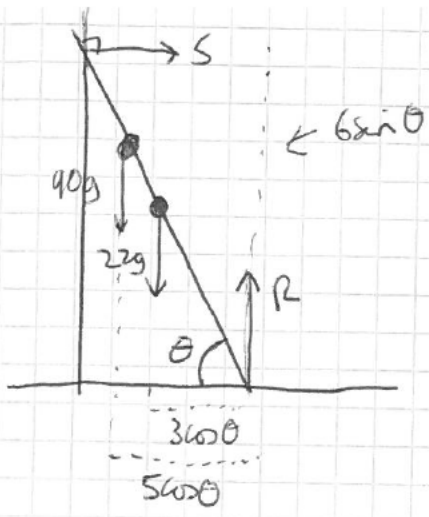
$$v = \sqrt{\frac{2 \times (247.5 + 5390\sqrt{3})}{55}}$$

$$= 18.6677$$

$$v = 18.7 \text{ ms}^{-1}$$

(c) No air resistance

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(a) Resolve vertically

$$R = 90g + 22g = 112g$$

$$F = \mu R = 0.6 \times 112g = 67.2g$$

$$= 658.56$$

$$= \underline{659} \text{ (3sf)}$$

(b) Take moments about A

Moments must "balance".

$$S \times 6 \sin \theta = 22g \times 3 \cos \theta + 90g \times 5 \cos \theta$$

$$S = F = 67.2g$$

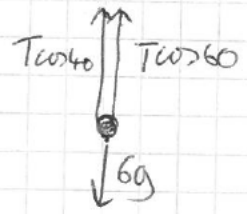
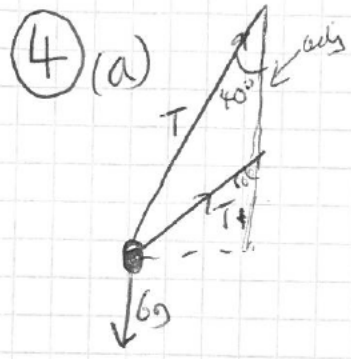
$$\Rightarrow 6 \times 67.2g \sin \theta = 516g \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{516g}{403.2g} = \frac{215}{168}$$

$$\Rightarrow \tan \theta = \frac{215}{168}$$

$$\Rightarrow \theta = 57.996$$

$$\theta = \underline{\underline{52.0^\circ}} \text{ (3sf)}$$



$$T(\cos 40 + \cos 60) = 6g$$

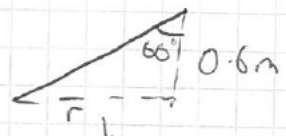
$$T = \frac{6g}{\cos 40 + \cos 60} = 46.4439$$

$T = 46.4$ (3sf) as required.

(b) $F = ma$
 $= \frac{mv^2}{r}$

$$F = T \sin 40 + T \sin 60$$

$$T(\sin 40 + \sin 60) = \frac{mv^2}{r}$$



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 60 = \frac{\text{opp}}{0.6}$$

$$r = 0.6 \tan 60$$

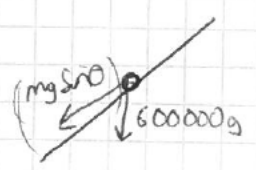
save T into calc. memory to avoid rounding error.

$$\Rightarrow \frac{T(\sin 40 + \sin 60)}{0.6 \tan 60} = \frac{6v^2}{r}$$

$$\Rightarrow v = 3.48387$$

$$v = 3.48 \text{ m s}^{-1}$$

5) Constant speed $\Rightarrow a=0 \Rightarrow$ Net force = 0.



$$\begin{aligned} \text{Total force opposing motion} &= mg \sin \theta + 200000 \\ &= 600000 \times 9.8 \times \frac{1}{40} + 200000 \\ &= 347000 \text{ N} \end{aligned}$$

F_D (Driving force) \Rightarrow must be 347000 ($F=0$)

$$F_D = \frac{P}{v} \Rightarrow P = F_D v = 347000 \times 24 = 8328000 \text{ W} = \underline{\underline{8328 \text{ kW}}}$$

⑥ $l = 1.2\text{m}$ $\lambda = 180\text{N}$

(a) $x = 2 - 1.2 = 0.8$ $EPE = \frac{\lambda x^2}{2} = \frac{180 \times 0.8^2}{2 \times 1.2} = 48\text{J}$

(b) No wd against friction
String will be slack so no EPE

All of the 48 J is converted to E_k

$$\frac{1}{2}mv^2 = 48$$

$$\frac{1}{2} \times 5 v^2 = 48$$

$$\Rightarrow v = \sqrt{\frac{2 \times 48}{5}} = \frac{4\sqrt{30}}{5} = 4.38178$$

$$= 4.38\text{ms}^{-1}$$

(3sf)

(c) Just as it reaches the wall so $E_k = 0$

All of the 48 J is "lost" as wd against friction.

$$Wd = F \times s \Rightarrow F = \frac{Wd}{s}$$

$$\frac{48}{2} = F = \underline{24\text{N}}$$

$$F = \mu R = 24\text{N}$$

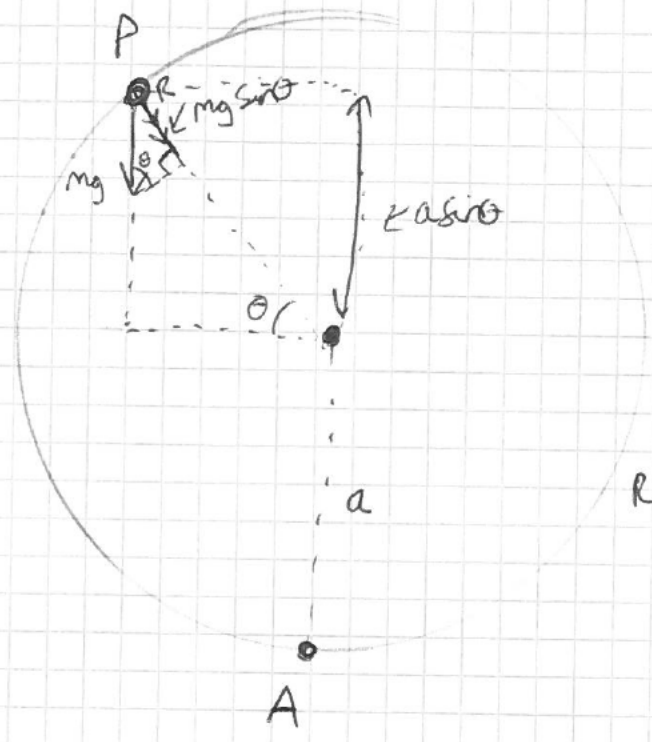
$$R = mg = 5 \times 9.8 = 49\text{N}$$

$$24 = \mu R = 49\mu$$

$$\mu = 0.4897959$$

$$\mu = 0.490 \text{ (3sf)}$$

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Net force
(radially)
 $= R + mg \sin \theta$

$$R + mg \sin \theta = \frac{mv^2}{r} \quad (1)$$

Consider E_k at P.

Gained $GPE = mg \Delta h$

gained height (from A)
is $a + a \sin \theta$
 $= a(1 + \sin \theta)$

$$\Delta GPE = mga(1 + \sin \theta)$$

So $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \sin \theta)$

\downarrow E_k at P \downarrow E_k at A

$$\Rightarrow v^2 = u^2 - 2ga(1 + \sin \theta)$$

$$(1) \quad R + mg \sin \theta = \frac{m}{a} (u^2 - 2ga(1 + \sin \theta))$$

$\downarrow \frac{mv^2}{r}$

$$R + mg \sin \theta = \frac{mu^2}{a} - 2mg(1 + \sin \theta)$$

$$R + mg \sin \theta = \frac{mu^2}{a} - 2mg - 2mg \sin \theta$$

$$R = \frac{mu^2}{a} - 2mg - 3mg \sin \theta$$

as required.

(b) $u^2 = 3ag$

When it leaves the surface

$R=0$

$$0 = \frac{mu^2}{a} - 3mg \sin \theta - 2mg$$

$$0 = \frac{3ag}{a} - 3g \sin \theta - 2g$$

$$0 = 3 - 3 \sin \theta - 2 \quad \Rightarrow \quad 0 = 1 - 3 \sin \theta$$

$$\sin \theta = \frac{1}{3}$$

$$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^\circ$$

19.5° (3sf)

⑧ (a) $F_r = -7mv^{\frac{3}{2}}$

use $F=ma \Rightarrow a = \frac{F}{m}$

$a = -7v^{\frac{3}{2}}$

$a = \frac{dv}{dt} = -7v^{\frac{3}{2}}$

(b) $\frac{dv}{dt} = -7v^{\frac{3}{2}}$

Separate variables.

$\int \frac{1}{v^{\frac{3}{2}}} dv = -\int 7 dt \Rightarrow \int v^{-\frac{3}{2}} dv = -\int 7 dt$

$\Rightarrow \frac{v^{-\frac{1}{2}}}{-\frac{1}{2}} = -7t + C$

$-2v^{-\frac{1}{2}} = -7t + C \quad (x-1)$

$\frac{2v}{\sqrt{v}} = 7t + d$

$t=0$ $v=9$ $\Rightarrow \frac{2}{\sqrt{9}} = 0 + d$

so $d = \frac{2}{3}$

$\frac{2}{\sqrt{v}} = 7t + \frac{2}{3}$

$\frac{\sqrt{v}}{2} = \frac{1}{7t + \frac{2}{3}} \Rightarrow \sqrt{v} = \frac{2}{7t + \frac{2}{3}} \quad \frac{\times 3}{\times 3}$

$\sqrt{v} = \frac{6}{37t + 2}$

$\Rightarrow v = \frac{36}{(2+37t)^2}$

(c) use $\sqrt{v} = \frac{6}{37t+2}$

$v=4$ $\sqrt{4} = \frac{6}{37t+2} \Rightarrow 2 = \frac{6}{37t+2} \Rightarrow 3 = 37t+2$

$\Rightarrow 1 = 37t$ so $t = \frac{1}{37}$